Summary of Lecture 10

- Second quantisation: represent many-particle state by the occupation numbers $\{N_\alpha\}$ of single-particle states $\{\varphi_\alpha(\mathbf{r})\}$

- Creation/annihilation operators

$$\hat{a}_\alpha^\dagger |N_0, N_1, \ldots, N_\alpha, \ldots\rangle \to \sqrt{N_\alpha + 1}\, |N_0, N_1, \ldots, N_\alpha + 1, \ldots\rangle$$

$$\hat{a}_\alpha |N_0, N_1, \ldots, N_\alpha, \ldots\rangle \to \sqrt{N_\alpha}\, |N_0, N_1, \ldots, N_\alpha - 1, \ldots\rangle$$

- Bosons/Fermions: Commutation/Anti-Commutation relations

$$[\hat{a}_\alpha, \hat{a}_\beta^\dagger]_\mp = \delta_{\alpha,\beta} \qquad [\hat{a}_\alpha, \hat{a}_\beta]_\mp = [\hat{a}_\alpha^\dagger, \hat{a}_\beta^\dagger]_\mp = 0$$

- Bosons $$|N_0, N_1, \ldots\rangle = \prod_{\alpha=0}^{\infty} \frac{(\hat{a}_\alpha^\dagger)^{N_\alpha}}{\sqrt{N_\alpha!}} |0,0,0,\ldots\rangle$$

- Fermions $$|N_0, N_1, \ldots\rangle = (\hat{a}_0^\dagger)^{N_0}(\hat{a}_1^\dagger)^{N_1}(\hat{a}_2^\dagger)^{N_2}\ldots |0,0,0,\ldots\rangle$$

This Lecture (11)

- Position Basis, Operators, Correlations & Interactions

Summary of Lecture 11

- Boson/Fermion Field Operators (position basis)

$$[\hat{\psi}(\vec{r}), \hat{\psi}^\dagger(\vec{r}')]_\mp = \delta(\vec{r} - \vec{r}') \qquad [\hat{\psi}(\vec{r}), \hat{\psi}(\vec{r}')]_\mp = [\hat{\psi}^\dagger(\vec{r}), \hat{\psi}^\dagger(\vec{r}')]_\mp = 0$$

- Density operator $\hat{\rho}(\vec{r}) = \hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r})$

- Single-particle density matrix $g(\vec{r}, \vec{r}') = \langle \hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r}') \rangle$

- Density-density correlations

$$\langle \hat{\rho}(\vec{r})\hat{\rho}(\vec{r}') \rangle = \langle \hat{\psi}^\dagger(\vec{r})\hat{\psi}^\dagger(\vec{r}')\hat{\psi}(\vec{r}')\hat{\psi}(\vec{r}) \rangle + \delta(\vec{r} - \vec{r}')\langle \hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r}) \rangle$$

$$= \qquad \langle : \hat{\rho}(\vec{r})\hat{\rho}(\vec{r}') : \rangle \qquad + \delta(\vec{r} - \vec{r}')\langle \rho(\vec{r}) \rangle$$

Next Lecture (12)

- Bose-Hubbard Model
- Bogoliubov Transformation
- Interference of Condensates