Summary of Lecture 2

- Spin in a field $H(t) = \overrightarrow{H}(t) \cdot \vec{S}$

  Heisenberg picture $\dfrac{\mathrm{d}}{\mathrm{d}t}\vec{S}(t) = \dfrac{1}{\hbar}\overrightarrow{H}(t) \times \vec{S}(t)$

  Rabi Oscillations $\overrightarrow{H}(t) = (H_R \cos \omega t, H_R \sin \omega t, H_0)$

  Transform to rotating frame $\overrightarrow{H}_{\mathrm{Rabi}} = (H_R, 0, H_0 - \hbar\omega)$

- Adiabatic approximation (Schrödinger Picture)

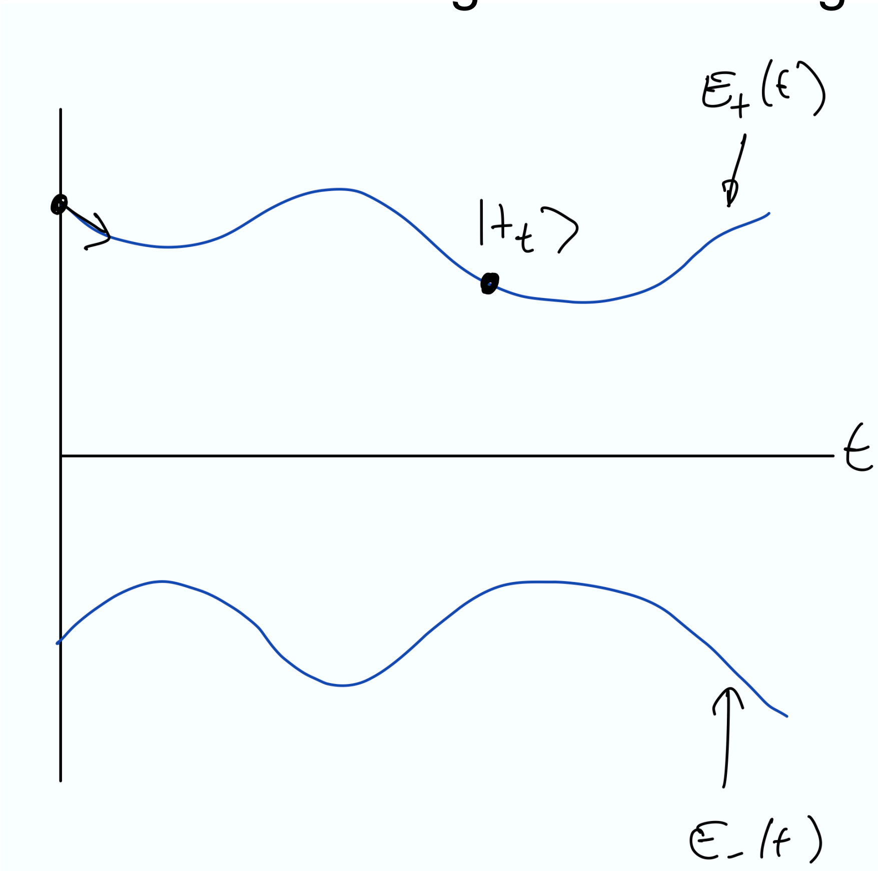
This Lecture (3)

- Landau-Zener Tunneling
- Berry Phase

Adiabatic Approximation: Recap

$H(t) = \overrightarrow{H}(t) \cdot \vec{S}$

Instantaneous eigenvectors / eigenvalues



$$|\Psi(t)\rangle = c_+(t)\,|+_t\rangle + c_-(t)\,|-_t\rangle$$

no transitions between $|+_t\rangle, |-_t\rangle$
if $H(t)$ changes slowly

$$c_\pm(t) = \mathrm{e}^{\left[-\frac{\mathrm{i}}{\hbar}\int_0^t E_\pm(t')\right]\mathrm{d}t'}\,c_\pm(0)$$

e.g. Born-Oppenheimer approximation
(nuclei move slowly compared to electrons)

Adiabatic Approximation: Recap

How slowly?

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} c_+ \\ c_- \end{pmatrix} = \begin{pmatrix} E_+(t) & i\hbar\frac{\langle +_t|\dot{H}|-_t\rangle}{E_+ - E_-} \\ -i\hbar\frac{\langle -_t|\dot{H}|+_t\rangle}{E_+ - E_-} & E_-(t) \end{pmatrix}\begin{pmatrix} c_+ \\ c_- \end{pmatrix}$$

$$\left|\hbar\frac{\langle +_t|\dot{H}|-_t\rangle}{E_+ - E_-}\right| \ll |E_+ - E_-|$$

- Avoid degeneracy
- Semiclassical, $\hbar$ small (action, $S/\hbar \gg 1$)

Berry Phase: Applications

— Aharonov-Bohn Effect

— Vibrations of molecules

— "Topological Insulators"

Integer quantum Hall effect (2D) [Thouless *et al*, 1982]

$$\int_{\mathrm{B.Z.}} B_n(\mathbf{k})\,\mathrm{d}^2\mathbf{k} = 2\pi C_n$$

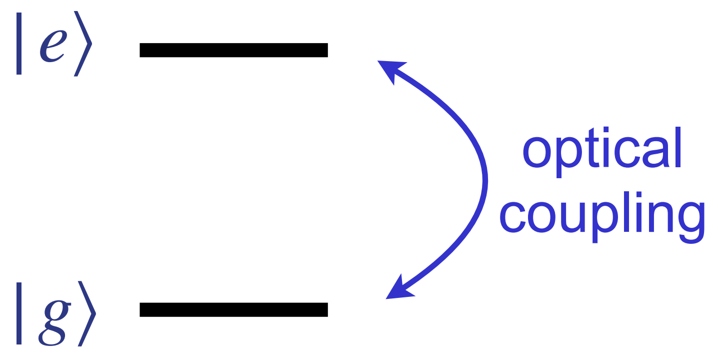
Integer "Chern number"

Synthetic Magnetic Field for Neutral Atoms

1) Optical fields lead to “dressed states”

$$|\psi\rangle = c_g|g\rangle + c_e|e\rangle$$

$$= \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2\ \mathrm{e}^{\mathrm{i}\phi} \end{pmatrix}$$



2) Spatial variations of $\theta(\mathbf{r}), \phi(\mathbf{r})$ — i.e. of the optical fields — give Berry potential & curvature

$$\mathbf{A} = -\hbar \sin^2\theta/2\,\nabla\phi$$

$$\mathbf{B}_{\mathrm{eff}} = \nabla\times\mathbf{A}$$

⇒ effective magnetic field for the positional motion

Summary of Lecture 3

- Landau Zener transitions $P(|-\rangle \to |-\rangle) \simeq \frac{\pi\Delta^2}{\hbar\beta}$ (when $\ll 1$)

- Berry Phase $c_\pm(t) = \mathrm{e}^{\left[-\frac{\mathrm{i}}{\hbar}\int_0^t E_\pm(t')\mathrm{d}t'\right]} e^{+\mathrm{i}\theta_{B,\pm}} c_\pm(0)$

$$\theta_{B,\pm}[\gamma] = -\int_\gamma \overrightarrow{A}_\pm \cdot \mathrm{d}\overrightarrow{H}$$ (depends on geometry of path, γ)

$$\overrightarrow{A}_\pm = -\mathrm{i}\langle \pm, \overrightarrow{H}|\frac{\mathrm{d}}{\mathrm{d}\overrightarrow{H}}|\pm, \overrightarrow{H}\rangle$$ gauge-dependent Berry potential

Next Lecture (4): Introduction to Path Integrals

- The propagator & Green's function