Summary of Lecture 3

- Landau Zener transitions $P(|-\rangle \to |-\rangle) \simeq \frac{\pi\Delta^2}{\hbar\beta}$ (when $\ll 1$)

- Berry Phase $c_\pm(t) = \mathrm{e}^{\left[-\frac{\mathrm{i}}{\hbar}\int_0^t E_\pm(t')\mathrm{d}t'\right]} e^{+\mathrm{i}\theta_{B,\pm}} c_\pm(0)$

$\theta_{B,\pm}[\gamma] = -\int_\gamma \vec{A}_\pm \cdot \mathrm{d}\vec{H}$ (depends on geometry of path, γ)

$\vec{A}_\pm = -\mathrm{i}\langle \pm, \vec{H}|\frac{\mathrm{d}}{\mathrm{d}\vec{H}}|\pm, \vec{H}\rangle$ gauge-dependent Berry potential

This Lecture (4): Introduction to Path Integrals

- The propagator & Green's function

Introduction to Path Integrals

- Schrödinger Picture

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle$$

- Heisenberg Picture — state, $|\Psi(0)\rangle$, time-independent

$$O(t) = U^{\dagger}(t)OU(t) \qquad \frac{\mathrm{d}}{\mathrm{d}t}O(t) = \frac{\mathrm{i}}{\hbar}\left[H(t), O(t)\right]$$

- Feynman Path integral

a method to calculate the *propagator*

$$K(\vec{r},t|\vec{r}',t') \equiv \theta(t-t')\langle\vec{r}|U(t-t')|\vec{r}'\rangle$$

step function $\theta(t-t')$ position eigenstate $|\vec{r}\rangle$

Summary of Lecture 4

- Propagator

$$K(\vec{r},t\,|\,\vec{r}',t') \equiv \theta(t-t')\langle \vec{r}\,|\,U(t-t')\,|\,\vec{r}'\rangle$$

- Relation to Green's functions

$$\left[\mathrm{i}\hbar\frac{\partial}{\partial t} - H\right] K(\vec{r},t\,|\,\vec{r}',t') = \mathrm{i}\hbar\delta(t-t')\delta(\vec{r}-\vec{r}')$$

$$K(\vec{r},t\,|\,\vec{r}',t') = 0 \quad (t < t')$$

- Free particle

$$K(\vec{r},t\,|\,\vec{r}',t') \equiv \theta(t-t')\left[\frac{m}{2\pi\mathrm{i}\hbar(t-t')}\right]^{3/2} \exp\left[-\frac{m\,|\,\vec{r}-\vec{r}'\,|^2}{2\mathrm{i}\hbar(t-t')}\right]$$

Next Lecture (5)

- The Feynman Path Integral