Summary of Lecture 5

- Propagator

$$K(\vec{r}_f, t_f | \vec{r}_i, t_i) = \int \mathscr{D}\vec{r}(t) \exp\left[\frac{\mathrm{i}}{\hbar} S[\vec{r}(t)]\right]$$

Action $$S[\vec{r}(t)] \equiv \int_{t_i}^{t_f} L[\vec{r}(t), \dot{\vec{r}}(t)]\, \mathrm{d}t$$

Lagrangian $$L[\vec{r}(t), \dot{\vec{r}}(t)] \equiv \frac{m\dot{\vec{r}}^2}{2} - V(\vec{r})$$

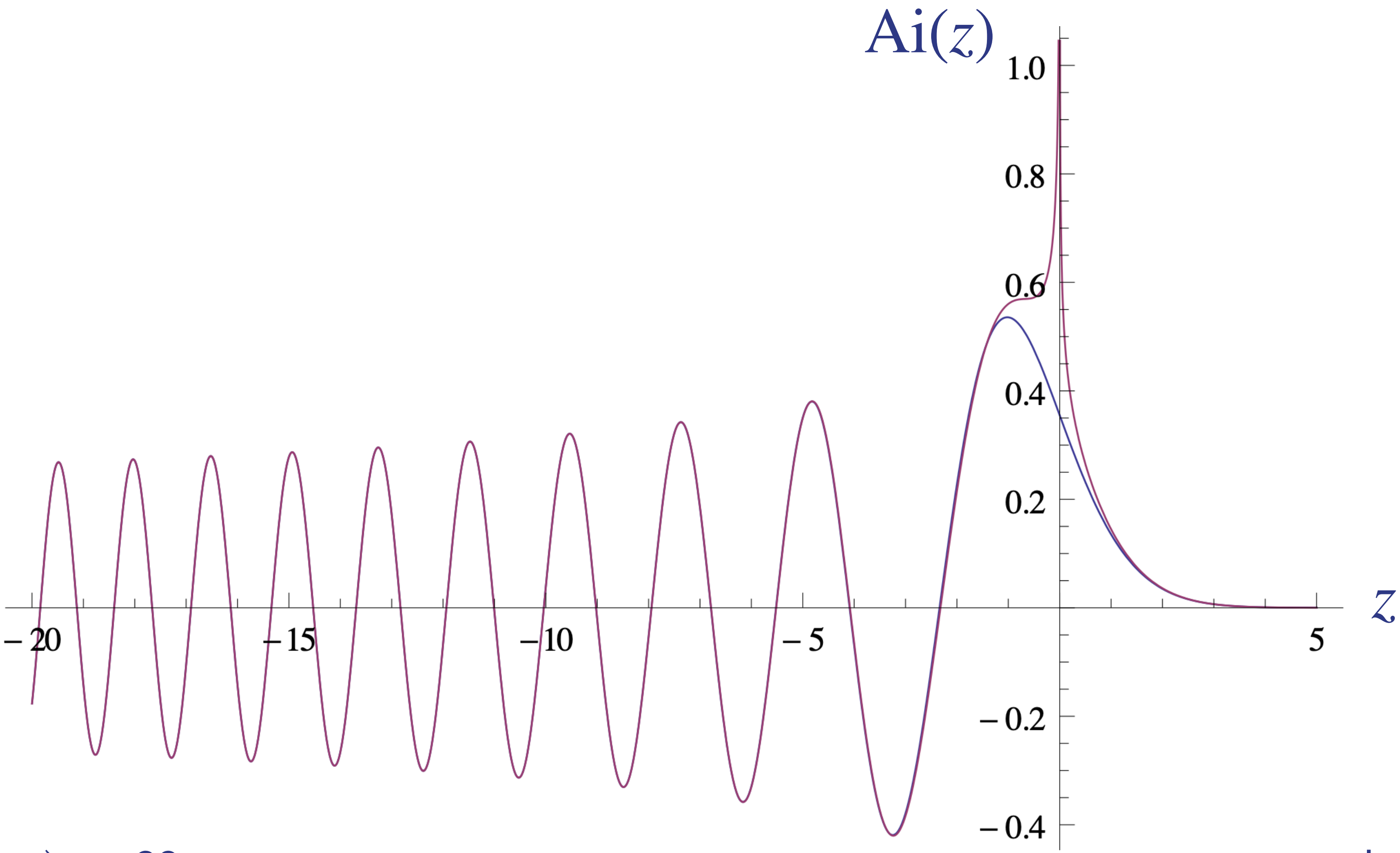
- Propagator for the SHO

- The Semiclassical limit

This Lecture (6)

- The JWKB method and Stationary Phase

Airy Function and JWKB Approximation



$z \to -\infty$

$z \to +\infty$

$$\frac{1}{\pi^{1/2}|z|^{1/4}} \sin\left(\frac{2}{3}|z|^{3/2} + \frac{\pi}{4}\right)$$

$$\frac{1}{2\pi^{1/2}z^{1/4}} \exp\left(-\frac{2}{3}z^{3/2}\right)$$

JWKB Connection Formulae

$x > a$ classically forbidden

$$\frac{2}{\sqrt{k(x)}} \cos\left[\int_x^a k(x')\,dx' - \frac{\pi}{4}\right] \iff \frac{1}{\sqrt{K(x)}} e^{-\int_a^x K(x')\,dx'}$$

$$\frac{1}{\sqrt{k(x)}} \sin\left[\int_x^a k(x')\,dx' - \frac{\pi}{4}\right] \iff -\frac{1}{\sqrt{K(x)}} e^{\int_a^x K(x')\,dx'}$$

$x < b$ classically forbidden

$$\frac{1}{\sqrt{K(x)}} e^{-\int_x^b K(x')\,dx'} \iff \frac{2}{\sqrt{k(x)}} \cos\left[\int_b^x k(x')\,dx' - \frac{\pi}{4}\right]$$

$$\frac{1}{\sqrt{K(x)}} e^{\int_x^b K(x')\,dx'} \iff -\frac{1}{\sqrt{k(x)}} \sin\left[\int_b^x k(x')\,dx' - \frac{\pi}{4}\right]$$

Summary of Lecture 6

- JWKB approximation

$$\psi(x) \sim \frac{1}{\sqrt{k(x)}} \mathrm{e}^{\pm \mathrm{i} \int^x k(x')\mathrm{d}x'} \qquad k(x) = \sqrt{2m[E - V(x)]}/\hbar$$

valid for $\frac{\mathrm{d}k}{\mathrm{d}x} \ll k^2 \qquad \hbar \frac{\mathrm{d}p}{\mathrm{d}x} \ll p^2$

- Connection formulae arise from evaluating

$$f(z) = \frac{1}{2\pi} \int_{\mathrm{contour}} \mathrm{e}^{\mathrm{i}kz + \mathrm{i}k^3/3} \mathrm{d}k$$

in the stationary phase approximation, valid for $z \to \pm \infty$

Next Lecture (7)

- Scattering Theory