Summary of Lecture 8

- First Born approximation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}_{\mathrm{Born}} = \left| \frac{m}{2\pi\hbar^2} \int \mathrm{d}\vec{r}' \, \exp\left(-\mathrm{i}\vec{q}\cdot\vec{r}'\right) V(\vec{r}') \right|^2$$

- Partial wave analysis: conserved angular momentum

$$f(\theta,\phi) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1)\, \mathrm{e}^{\mathrm{i}\delta_l} \sin\delta_l \, P_l(\cos\theta)$$

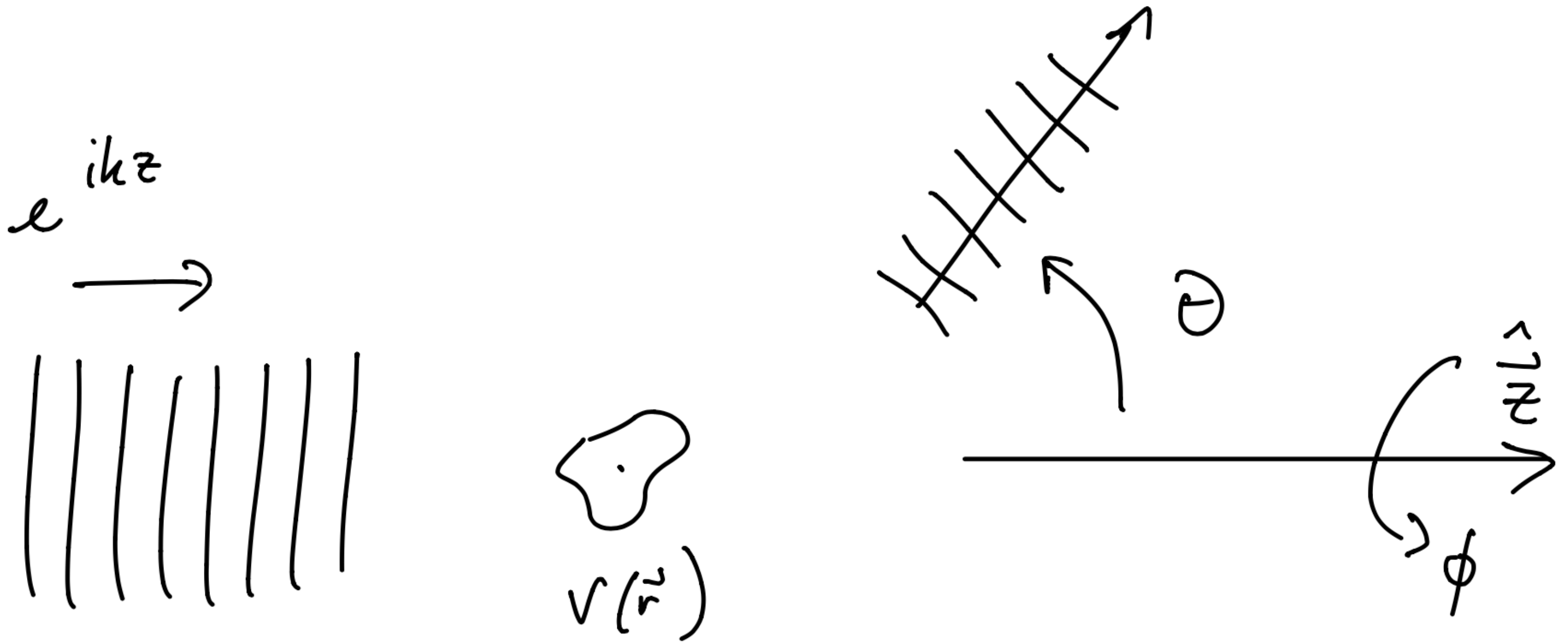
$$\sigma_{\mathrm{tot}} = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)\sin^2\delta_l = \frac{4\pi}{k} \mathrm{Im}\left[f(0)\right]$$

[Optical Theorem]

This Lecture (9)

- Low-energy scattering, Resonances and Bound States

Scattering in 3D



$$\Psi_k(\vec{r}) \xrightarrow[r\to\infty]{} \exp\left(\mathrm{i}kz\right) + \frac{f(\theta,\phi)}{r}\exp\left(\mathrm{i}kr\right)$$

Differential cross section
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|f(\theta,\phi)\right|^2$$

Spherical Symmetry $V(\vec{r}) = V(|\vec{r}|)$

- Partial wave analysis: conserved angular momentum

$$\Psi_k(\vec{r}) \xrightarrow{r\to\infty} \sum_{l=0}^{\infty} (2l+1)\mathrm{i}^l P_l(\cos\theta) \left[\mathrm{e}^{2\mathrm{i}\delta_l} \frac{\mathrm{e}^{\mathrm{i}(kr - l\pi/2)}}{2\mathrm{i}kr} - \frac{\mathrm{e}^{-\mathrm{i}(kr - l\pi/2)}}{2\mathrm{i}kr} \right]$$

phase shifts $\delta_l(k)$

$$f(\theta,\phi) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1)\, \mathrm{e}^{\mathrm{i}\delta_l} \sin\delta_l P_l(\cos\theta)$$

⇒ Differential cross section $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\theta)|^2$ etc.

Summary of Lecture 9

- s-wave scattering, $l = 0$, dominates at low energy

$$\sigma_{\text{tot}} \xrightarrow{k \to 0} \frac{4\pi}{k^2} \sin^2 \delta_{l=0}$$

- Scattering length $\delta_0 \to -ka \;\; (+n\pi)$ $\sigma_{\text{tot}} \to 4\pi a^2$

- Full calculation for a potential step (for $l = 0$)

[On resonance $\delta_0 \to (n + 1/2)\pi$ $\sigma_{\text{tot}} \to 4\pi / k^2$]

Next Lecture (10) ***Monday 24th Feb***

- Identical Particles and Second Quantization