Vortices and vortex interactions in exciton-polariton condensates

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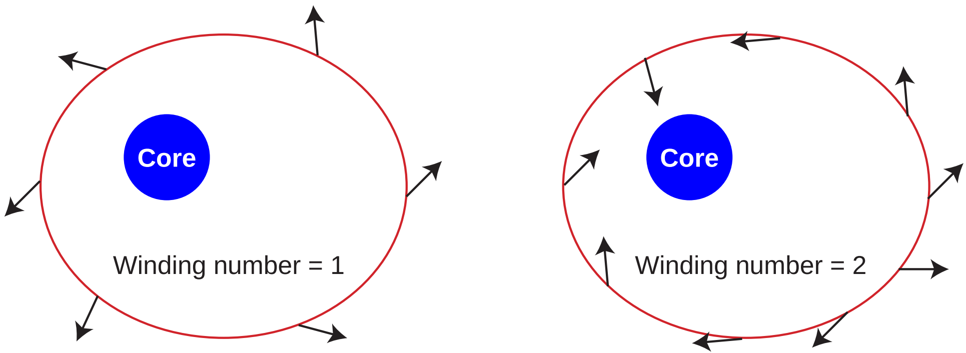
Outline

- Polarization of polariton superfluids
- Classification of vortices
- Half-vortices. The core texture. Half-vortices with strings
- Effects of TE-TM splitting
- Generation of vortex lattices. Scattering on disorder

Vortices and their winding numbers

In the usual XY-model:



In a system with multicomponent order parameter it can be possible to have additional winding numbers.

In polariton condensates/superfluids the order parameter is the polarization (complex) vector.
There are two winding numbers.

The Gross-Pitaevskii equation

The order parameter is two-dimensional complex vector $\vec{\psi}$.
The energy functional for polariton superfluid

$$H = H_{\rm kin} + H_{\Omega} + H_{\rm int} =$$
$$= \int d^2r \left\{ \vec{\psi}^* \cdot \left(-\frac{1}{2m^*}\Delta - \mu \right) \vec{\psi} + {\rm i}\Omega[\vec{\psi}^* \times \vec{\psi}] + \frac{1}{2}\left[U_0(\vec{\psi}^* \cdot \vec{\psi})^2 - U_1 \vec{\psi}^{*2}\vec{\psi}^2 \right] \right\},$$

where $\Omega = (1/2)g\mu_{\rm B}B$ is half of the Zeeman splitting of free polariton.

The Gross-Pitaevskii equation reads

$${\rm i}\frac{\partial \psi_i}{\partial t} = \frac{\delta H}{\delta \psi_i^*} = \left[-\frac{1}{2m^*}\Delta - \mu \right]\psi_i + {\rm i}\Omega\varepsilon_{ij}\psi_j + U_0\psi_j^*\psi_j\psi_i - U_1\psi_j\psi_j\psi_i^*,$$

where $\varepsilon_{xy} = -\varepsilon_{yx} = 1$, $\varepsilon_{xx} = \varepsilon_{yy} = 0$.

Linear polarization at zero magnetic field

Formation of linear polarization in polariton condensates [Le Si Dang *et al.*; Snoke *et al.*, 2006] arises due to the reduction of polariton-polariton repulsion energy $H_{\rm int}$:

$$H_{\rm int} = \frac{1}{2}\int d^2r \left\{(U_0 - U_1)(\vec{\psi}^* \cdot \vec{\psi})^2 + U_1|\vec{\psi}^* \times \vec{\psi}|^2\right\}.$$

Two interaction constants, $U_0 = AM_{\uparrow\uparrow}$ and $U_1 = A(M_{\uparrow\uparrow} - M_{\uparrow\downarrow})/2$, where $A = \pi R^2$ is the excitation spot area. Typically, $U_0/2 < U_1 < U_0$.

At a fixed concentration $n = (\vec{\psi}^* \cdot \vec{\psi})$ minimum of $H_{\rm int}$ is reached for

$$\vec{\psi}^* \times \vec{\psi} = 0 \;\Rightarrow \text{Linear polarization}$$

One can write

$$\vec{\psi}_{\rm lin} = \{\psi_x, \psi_y\} = \sqrt{n}\,{\rm e}^{{\rm i}\theta}\{\cos\eta, \sin\eta\},$$

so that the order parameter is defined by two angles, η and θ.

Note that the states η, θ and $\eta + \pi, \theta + \pi$ are identical.

Condensate polarization in magnetic field

The uniform free-energy density

$$H_{\rm uni}/A = -\mu n - 2\Omega S_z + \frac{1}{2}(U_0 - U_1)n^2 + 2U_1 S_z^2, \qquad \Omega = (1/2)g\mu_{\rm B}B.$$

where $n = (\vec{\psi}^* \cdot \vec{\psi})$ and $S_z = ({\rm i}/2)[\vec{\psi} \times \vec{\psi}^*]$.

Weak fields: $\Omega \leqslant \Omega_c = nU_1$.
Condensate is elliptically polarized with $\rho = 2S_z/n$ and

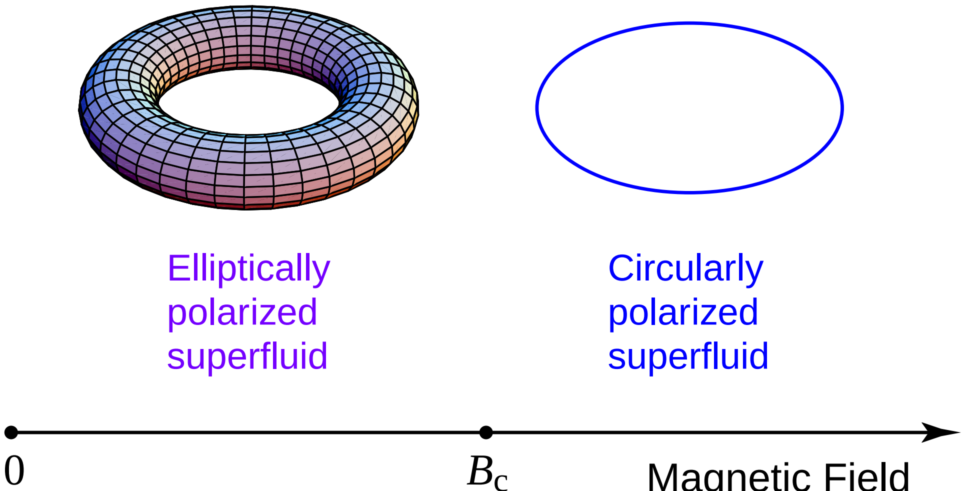
$$\mu_0 = (U_0 - U_1)n, \qquad S_z = \frac{\Omega}{2U_1}, \qquad H_{\rm min}/A = -\frac{1}{2}(U_0 - U_1)n^2 - \frac{\Omega^2}{2U_1}.$$

Strong fields: $\Omega > \Omega_c = nU_1$.
Condensate is fully circularly polarized, i.e., $S_z = n/2$, $(\rho = 1)$. Also

$$\mu = U_0 n - \Omega, \qquad H_{\rm min}/A = -\frac{1}{2}U_0 n^2,$$

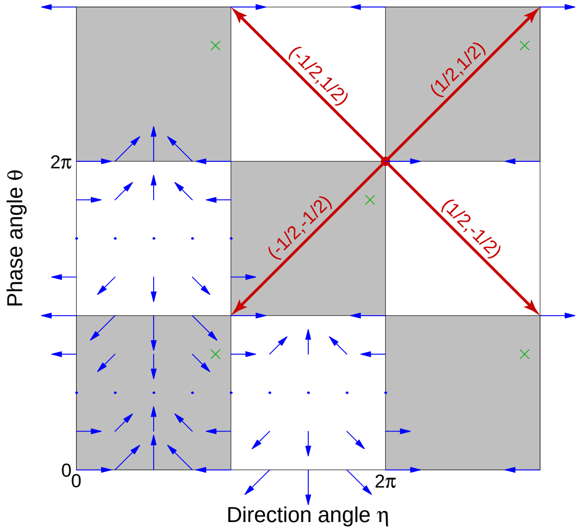
Quantum phase transition

The critical magnetic field corresponds to the quantum phase transition.
Topology of the order parameter space is changing at $B = B_c$:



The order parameter space

$$\vec{\psi}_{\rm elp} = e^{i\theta}\left\{f\cos\eta - ig\sin\eta,\, f\sin\eta + ig\cos\eta\right\}, \qquad f^2+g^2=1, \quad 2fg=\zeta.$$



The possible changes are:
$\eta \to \eta + 2\pi k$,
$\theta \to \theta + 2\pi m$.

Vortex carries two topological charges (winding numbers), (k, m).

Integer vortices:
$k, m = 0, \pm 1, \pm 2, \ldots$

Half-integer vortices:
$k, m = \pm 1/2, \pm 3/2, \ldots$

Half-vortices

Half-vortices in ^{3}He-A: G.E. Volovik and V.P. Mineev, (1976); M.C. Cross and W.F. Brinkman, (1977).

They appear due to combined spin-gauge symmetry:

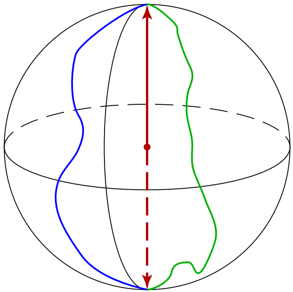
Spin quantization axis change $\vec{d} \to -\vec{d}$

Phase change $\theta \to \theta \pm \pi$

The superfluid velocity around the half-vortex $\vec{v}_s \propto \nabla\theta$ is a half of the superfluid velocity around the usual vortex with $\theta \to \theta \pm 2\pi$.

Half-vortex carries half-quantum of the superfluid current.

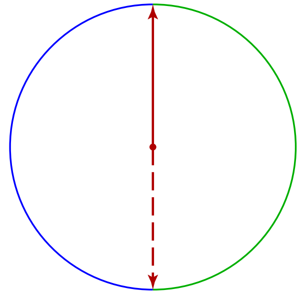
Why two winding numbers (k, m)?



Atomic spinor $s = 1$ condensates
(three-component)

3D real $\vec{d}$ and phase θ
Half-vortex: $\vec{d} \to -\vec{d}$, $\theta \to \theta + \pi$

All rotations $\vec{d} \to -\vec{d}$ are in the same homotopy class



Polariton pseudospin case
(two-component)

2D real $\vec{d}$ and phase θ
Half-vortex: $\vec{d} \to -\vec{d}$, $\theta \to \theta + \pi$

Clockwise and counterclockwise $\vec{d} \to -\vec{d}$ are topologically different

The half-vortex core

The core size $a = \hbar/\sqrt{2m^*\mu} \sim 1\ \mu\mathrm{m}$. For a basic half-vortex

$$\vec{\psi}_{\rm hv} = \sqrt{n}\,\left[\vec{A}(\phi) f(r/a) - \mathrm{i}\vec{B}(\phi) g(r/a)\right],$$

where the azimuthal dependencies are given by

$$\vec{A}(\phi) = \mathrm{e}^{im\phi}\{\cos(k\phi), \sin(k\phi)\},$$

$$\vec{B}(\phi) = \mathrm{sgn}(km)\mathrm{e}^{im\phi}\{\sin(k\phi), -\cos(k\phi)\},$$

and radial functions $f(r/a)$ and $g(r/a)$ are found from $\delta H/\delta\vec{\psi}^* = 0$:

$$f'' + \frac{1}{\xi}f' - \frac{1}{2\xi^2}(f-g) + \frac{1}{2}(\gamma-1)\omega g + [1 - f^2 - \gamma g^2]f = 0,$$

$$g'' + \frac{1}{\xi}g' - \frac{1}{2\xi^2}(g-f) + \frac{1}{2}(\gamma-1)\omega f + [1 - g^2 - \gamma f^2]g = 0,$$

where $\xi = r/a$, $\gamma = (U_0+U_1)/(U_0-U_1)$, and $\omega = \mathrm{sgn}(km)\Omega/nU_1$.
Conditions $f(0) = g(0)$, $f^2(\infty) + g^2(\infty) = 1$, $2f(\infty)g(\infty) = \omega$.

Half-vortex in circular polarizations

$$\vec{\psi}_{\rm elp} = \sqrt{n} e^{{\rm i}m\phi} \left\{ f\cos(k\phi) - {\rm i}g\sin(k\phi), f\sin(k\phi) + {\rm i}g\cos(k\phi) \right\} =$$
$$= \sqrt{n/2} \left([f + {\rm sgn}(km)g] e^{{\rm i}(m-k)\phi} |{\uparrow}\rangle + [f - {\rm sgn}(km)g] e^{{\rm i}(m+k)\phi} |{\downarrow}\rangle \right),$$

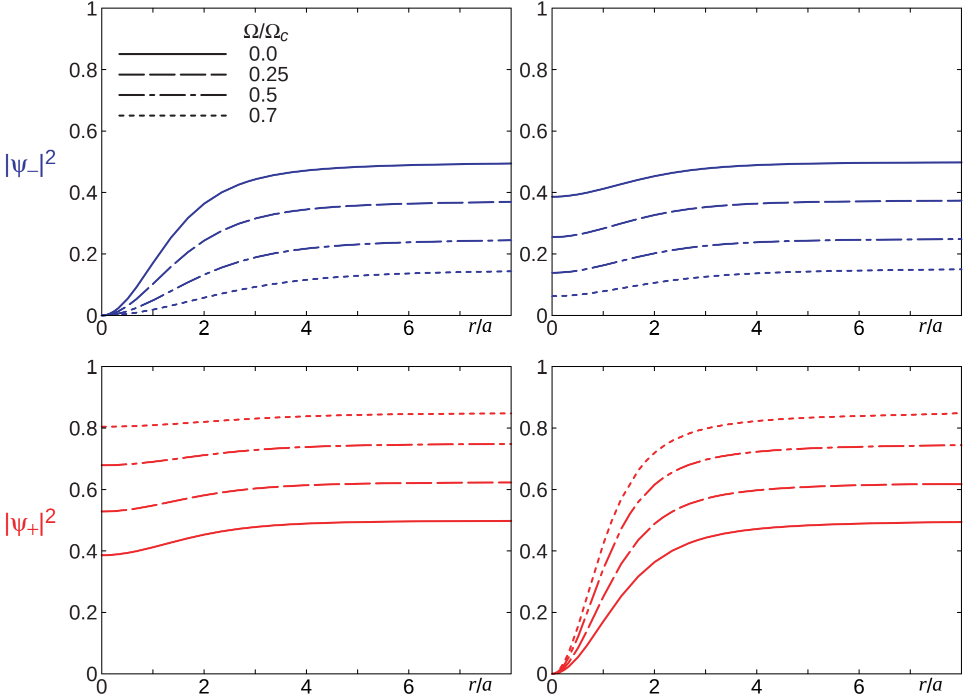
where

$$|{\uparrow}\rangle = \frac{1}{\sqrt{2}}\{1, {\rm i}\}, \qquad |{\downarrow}\rangle = \frac{1}{\sqrt{2}}\{1, -{\rm i}\},$$

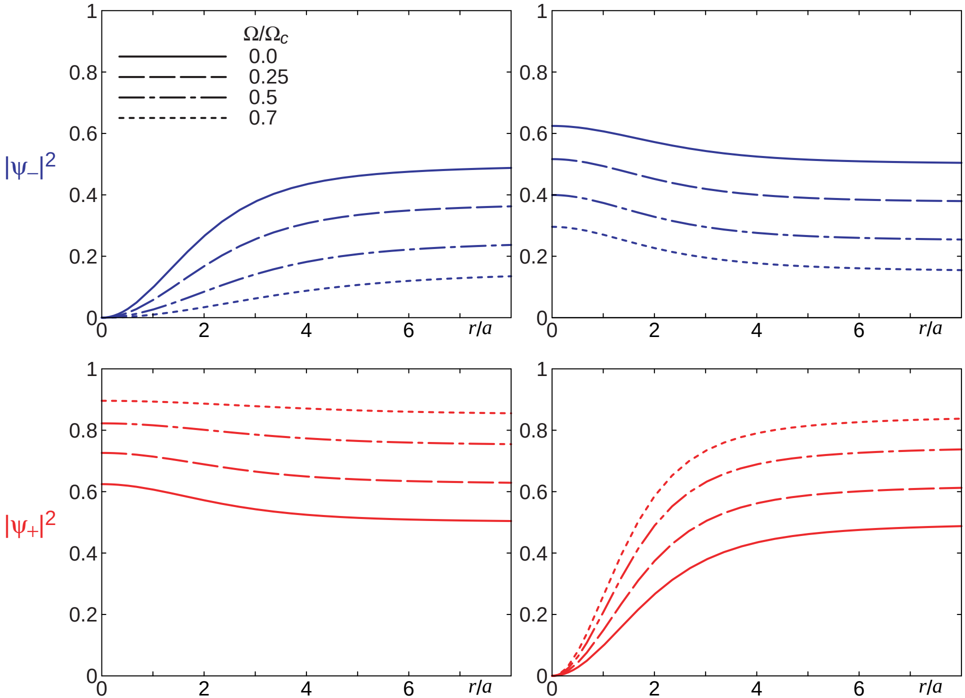
Right half-vortices: $km > 0$. Left-circular component becomes fully depleted and polarization is right-circular at $r = 0$.

Left half-vortices: $km < 0$. Right-circular component becomes fully depleted and polarization is left-circular at $r = 0$.

Atraction between spin-up and spin-down polaritons ($\gamma = 5$)

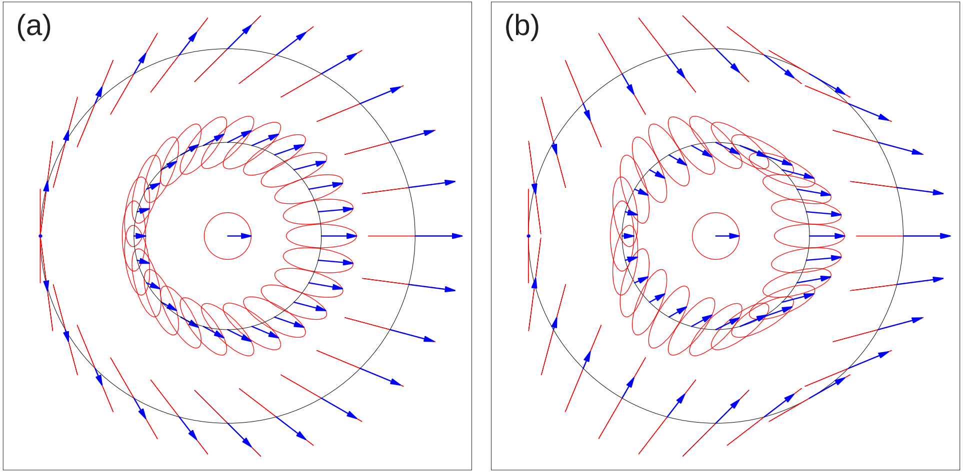


Repulsion between spin-up and spin-down polaritons ($\gamma = 2$)

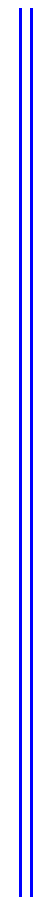


The polarization texture of half-vortex core

Showing $\mathrm{Re}\{\vec{\psi}e^{-\mathrm{i}\omega t}\}$, where $\omega = \omega_p + \mu$.



Two left half-vortices



Pair of left half-vortices

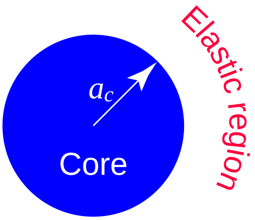
Permuted pair of left half-vortices

Half-vortex energies

$$\vec{\psi}_{\rm elp} = e^{i\theta}\left\{f\cos\eta - ig\sin\eta, f\sin\eta + ig\cos\eta\right\}, \qquad f^2+g^2=1, \quad 2fg=\zeta.$$

$$E = E_{\rm core} + E_{\rm el},$$

$$E_{\rm el} = \frac{1}{2}\rho_s \int d^2r \left[(\nabla\eta)^2 + (\nabla\theta)^2 - 2\zeta(\nabla\theta\cdot\nabla\eta)\right],$$



where $\rho_s = \hbar^2 n/m^*$, $\mu = n(U_0 - U_1)$, and $a_c = a_\pm = \hbar/[2m^*\mu(1\pm\zeta)]^{1/2}$.

Single vortex:

$$E_{\rm el}^{(\rm s)} = \frac{\pi}{2}\rho_s\left\{(1+\zeta)(k+m)^2\ln\left[\frac{R}{a_+}\right] + (1-\zeta)(k-m)^2\ln\left[\frac{R}{a_-}\right]\right\}.$$

The half-vortex energy $(\pi/2)\rho_s(1\pm\zeta)\ln(R/a_\pm) = (\pi/2)\rho_s^{(\pm)}\ln(R/a_\pm)$.

Vortex interactions

Vortex pair.

$$E_{\mathrm{el}}^{(\mathrm{p})} = \pi\rho_s[(k_1+k_2)^2+(m_1+m_2)^2]\ln(R/a) + 2\pi\rho_s(k_1k_2+m_1m_2)\ln(a/r).$$

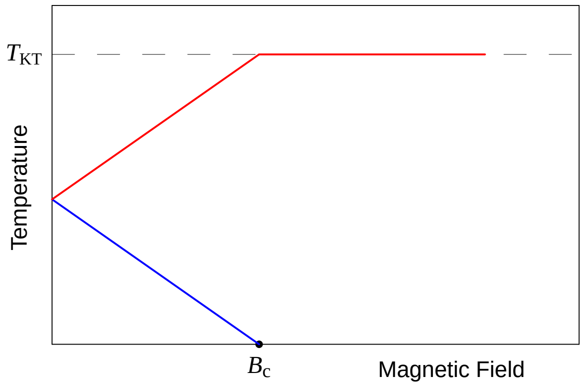
Right $(km > 0)$ half-vortices, $(1/2, 1/2)$ and $(-1/2, -1/2)$, interact with each other. But they don't interact with the left $(km < 0)$ half-vortices, $(1/2, -1/2)$ and $(-1/2, 1/2)$.

Berezinskii-Kosterlitz-Thouless transitions

Estimation of critical temperature.

$$F = E_{\rm el}^{\rm (s)} - T\ln(R/a)^2 = \left[\frac{\pi}{2}\rho_s^{(\pm)} - 2T\right]\ln(R/a), \qquad T_c^{(\pm)} = \frac{\pi}{4}\rho_s^{(\pm)}.$$

For $\Omega = 0$ the critical temperature $T_c^{(\pm)} = (1/2)T_{\rm KT}$, where $T_{\rm KT} = (\pi/2)\rho_s$.



Phase diagram in the HFP approximation: J. Keeling, arXiv:0801.2637

Polarization pinning. Half-vortices with strings

$$E_{\rm el} = \frac{1}{2}\rho_s \int d^2r \left\{ (\nabla\theta)^2 + (\nabla\eta)^2 + \epsilon[1 - \cos(2\eta)] \right\}$$

$$\Delta\theta = 0, \qquad \Delta\eta = \epsilon \sin(2\eta).$$

Without pinning ($\epsilon = 0$)

With pinning ($\epsilon > 0$)

In the case of pinning $E^{(p)}(r) \propto r$ for large distances.

Effects of TE-TM splitting

The kinetic energy density can be written as

$$T = \frac{\hbar^2}{2}\left\{\frac{1}{m_t}(\nabla_i\psi_j^*)(\nabla_i\psi_j) + \left(\frac{1}{m_l} - \frac{1}{m_t}\right)(\nabla_i\psi_i^*)(\nabla_j\psi_j)\right\}.$$

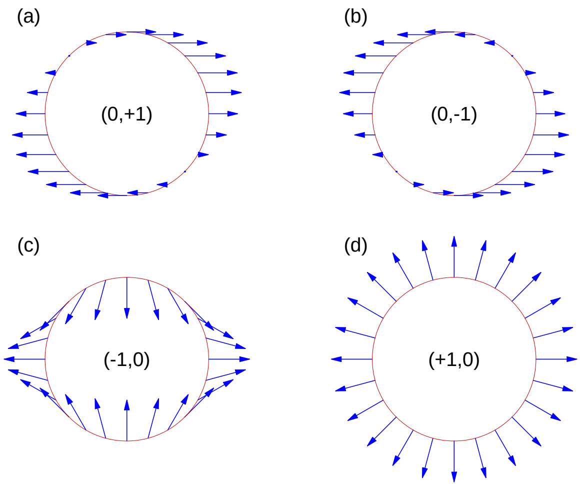
Elastic energy of a single vortex

$$E_{\mathrm{el}}^{(\mathrm{s})} = \frac{\pi\hbar^2 n}{2}\left[\left(\frac{1}{m_l} + \frac{1}{m_t}\right)(k^2 + m^2) + \left(\frac{1}{m_l} - \frac{1}{m_t}\right)(1 - m^2)\delta_{1,k}\right]\ln\left(\frac{R}{a}\right).$$

The main term is the same as without TE-TM splitting with

$$\frac{1}{m^*} = \frac{1}{2}\left(\frac{1}{m_l} + \frac{1}{m_t}\right).$$

Integer vortices



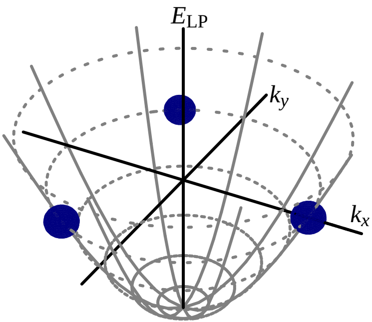
$$\text{Cases(a} - \text{c)}: \; E_s = \frac{\pi\hbar^2 n}{m^*}, \qquad \text{Case(d)}: \; E_s = \frac{\pi\hbar^2 n}{m_l}.$$

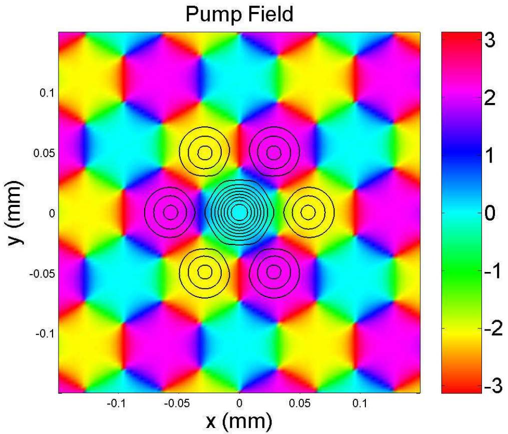
Since $(1,0) \rightarrow (1/2,-1/2)+(1/2,1/2)$, these half-vortices start to interact.

Three beams excitation

Vortices in singular optics: J.F. Nye and M.V. Berry (1974).
Several beam excitation: K. O'Holleran, M.J. Padgett, M.R. Dennis (2006)

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$





$$\mathrm{i}\frac{\partial \psi_\sigma}{\partial t} = \hat{T}\psi_\sigma - \frac{\mathrm{i}}{2\tau}\psi_\sigma + U_0|\psi_\sigma|^2\psi_\sigma + (U_0 - 2U_1)|\psi_{-\sigma}|^2\psi_\sigma + f_\sigma(\vec{r},t), \qquad \sigma = \pm$$

Evolution in linear case (1)

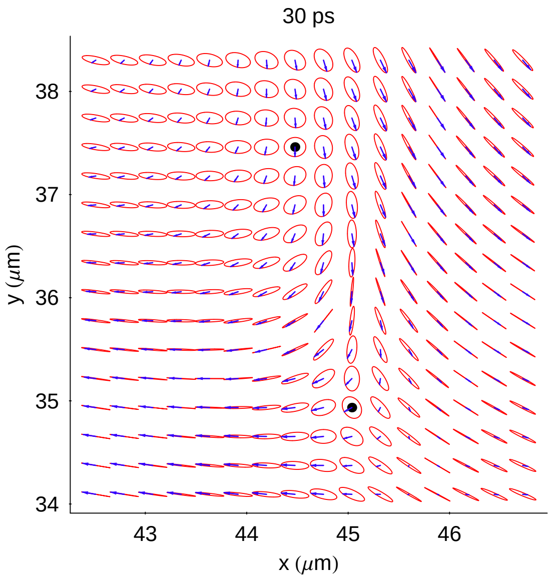
Evolution in linear case (2)

Nonlinear evolution

Half-vortex lattices (three TE beams)

One pulse with disorder potential

Scattering of elliptically polarized wave



Conclusions

- Vortices in polariton condensates in planar semiconductor microcavities carry two winding numbers (k, m). These numbers can be either integer or half-integer simultaneously.
- Pinning of polarization results in appearance of strings attached to half-vortices. An account for TE-TM splitting results in the interaction between left and right half-vortices.
- Vortices can appear when three or more coherent waves with different directions of their wave vectors overlap. This allows vortices to be injected into polariton condensates directly by overlapping several beams, or by using a single beam and exploiting the scattering of polaritons on disorder.