# Lecture 6
# Optics in Semiconductors. Excitons

Optics, i.e. probing the properties of a substance by means of light or another high frequency electromagnetic radiation, occupies an important place among the experimental methods of condensed matter physics. It differs from conventional d.c. and radio-frequency transport measurements in that it does not use any leads. It may consist in sending and/or receiving beams of light. Alternatively, a sample may be placed into a high-frequency cavity and studied how the presence of the sample changes the cavity quality factor. We will focus on beams in this lecture, while in Lecture 8 we will discuss the methods, which involve measuring the quality factors of high-frequency coils and cavities.

Optical experiments can be divided in the following categories:

- Measurements of transmission, reflection and adsorption coefficients as functions of frequency for different polarizations.
- Spectroscopy of photo-luminescence dependent on conditions (frequency and polarization) of photo-excitation.
- Coherent and combined light scattering
- Photo-voltaic phenomena (electric current generated by light).
- Non-linear optics, high harmonics generation, super-radiation and lasing.

All these phenomena occur in different substances. It is important to choose the conditions, which allow to make optics a real tool of investigating semiconductors.

## Dielectric function

Light propagation through a medium is described by the Maxwell equations, which have the following form (for dielectrics):

$$\text{div } \mathbf{B} = 0, \qquad \text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \ \mathbf{B}}{\partial t}; \tag{1}$$

$$\text{div } \mathbf{D} = 0, \qquad \text{curl } \mathbf{B} = \frac{1}{c}\frac{\partial \ \mathbf{D}}{\partial t}, \tag{2}$$

where the linear relation between the strength of electric field $\mathbf{E}$ and induction $\mathbf{D}$ is:

$$D_i(t, \mathbf{r}) = E_i(t, \mathbf{r}) + \int_{-\infty}^{t} dt' \int d\mathbf{r}' f_{ij}(|\mathbf{r} - \mathbf{r}'|, t - t') E_j(t', \mathbf{r}'). \tag{3}$$

Taking the Fourier transform of Eq (3), one gets

$$D_i(\omega, \mathbf{k}) = \epsilon_{ij} \ (\omega) \ E_j(\omega, \mathbf{k}); \tag{4}$$

$$\epsilon_{ij} \ (\omega, \mathbf{k}) = \delta_{ij} + \int_{-\infty}^{0} dt \ \int d\mathbf{r} \ f_{ij}(t, |\mathbf{r}|) \ e^{-i\omega t + i\mathbf{kr}}, \tag{5}$$

where $\epsilon(\omega, \mathbf{k})$ is the dielectric function. Eq (5) means that the dielectric function $\hat{\epsilon}(\omega)$ is an analytic function of frequency $\omega$ in the upper half plane (Im $\omega > 0$). Since the response function $f(t, \mathbf{r})$ is real at real $t$ and $\mathbf{r}$, the dielectric function $\hat{\epsilon}$ obeys the condition:

$$\hat{\epsilon}(-\omega, -\mathbf{k}) = \hat{\epsilon}^*(\omega, \mathbf{k}). \tag{6}$$

Let us forget for a moment about the dependence of the dielectric function $\epsilon$ on the wave vector of $\mathbf{k}$ (space dispersion). The complex function $\epsilon(\omega)$ of the real frequency $\omega$ has, therefore, an even in $\omega$ real part $\epsilon_1(\omega) = \mathrm{Re}\ \epsilon$ and an odd in $\omega$ imaginary part $\epsilon_2(\omega) = \mathrm{Im}\ \epsilon$, which correspond to refraction and adsorption of light respectively. The analytic character of the dielectric function $\epsilon(\omega)$ means that there are dispersion relations between its real and imaginary parts (*Kramers-Kronig relations*):

$$\epsilon_1(\omega) = 1 + \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\epsilon_2(x)}{x-\omega}\ dx = 1 + \frac{2}{\pi}\int_0^{\infty} \frac{x\ \epsilon_2(x)}{x^2-\omega^2}\ dx; \quad (7)$$

$$\epsilon_2(\omega) = -\ \frac{1}{\pi}\int \frac{\epsilon_1(x)-1}{x-\omega}\ dx = -\ \frac{2\omega}{\pi}\int_0^{\infty} \frac{\epsilon_1(x)-1}{x^2-\omega^2}\ dx. \quad (8)$$

For instance, if

$$\epsilon_2(\omega) = \gamma\ \frac{\Gamma^2}{(\omega-\omega_0)^2+\Gamma^2},$$

then it follows from Eq (7) that

$$\epsilon_1(\omega) = 1 - \gamma\ \frac{\Gamma(\omega-\omega_0)}{(\omega-\omega_0)^2+\Gamma^2},$$

i.e. absorption lines are accompanied by anomalous dispersion. This allows to use the Kramers-Kronig relations to extract the adsorption coefficient from from the measurements of transmission/reflection.

Light propagates through a medium with the speed $c_1 = c/\tilde{n}$, where $c$ is the speed of light in vacuum and $\tilde{n} = n + ik$ is the complex refraction index, while $n$ is the real refraction index and $k$ is $k$ is the extinction coefficient. The dielectric function $\epsilon$ is equal to the square of the complex refraction index $\tilde{n}$, which gives:

$$\epsilon_1 = n^2 - k^2, \qquad \epsilon_2 = 2\ n\ k. \tag{9}$$

The refraction index $n$ and extinction coefficient $k$ are, usually, determined from the reflection of light incident from vacuum. The reflection coefficient $R$ may be expressed through the coefficients $n$ and $k$:

$$R = \left|\frac{\tilde{n}-1}{\tilde{n}+1}\right|^2 = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2} \approx \frac{(n-1)^2}{(n+1)^2}, \tag{10}$$

where the last equality is valid in the case of weak absorption. The dielectric function $\epsilon_{ij}$ of a transparent crystal is usually a symmetric tensor

$$\epsilon_{ij} = \epsilon_{ji},$$

which means that light, linearly polarized along certain directions, propagates through the crystal keeping its polarization. This is not so if the time inversion symmetry is broken, which occurs in the presence of external magnetic field $\mathbf{B}$ (*Faraday effect*), or in magnetically ordered dielectrics (*Anomalous Faraday effect*). In all these cases a vector of gyration $\mathbf{G}$ (proportional either to the strength of magnetic field $\mathbf{B}$ or to the spontaneous magnetization $\mathbf{M}$) may be constructed and the dielectric function takes the form:

$$\epsilon_{ij} = \epsilon_{ij}^{(s)} + d_{ijk}\ G_k, \tag{11}$$

where $\epsilon_{ij}^{(s)}$ is a symmetric tensor (proportional to $\delta_{ij}$ in the case of an isotropic medium) and $\hat{d}$ is an anti-symmetric tensor( antisymmetric tensor $e_{ijk}$ for an isotropic medium). The plane of polarization for a linearly polarized light, propagating along the gyration vector $\mathbf{G}$, rotates (*Faraday effect*).

In materials without the inversion center, the space dispersion of the dielectric function also leads to rotation of the polarization plane. In the case of isotropic medium without the inversion centre, the dielectric function has the form:

$$\epsilon_{ij}(\omega, \mathbf{k}) = \epsilon^{(s)}\delta_{ij} + i\ G\ e_{ijl}\ k_l, \tag{12}$$

which shows that, irrespectively of the direction of propagation of a beam of linearly polarized light, its polarization plane rotates by $2\pi Gk/\epsilon^{(s)}$ over a unit length of its path.

## Dielectric function of free electron gas

In the presence of external electric field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{-i\omega t}$, a free electron moves, obeying the Newton law:

$$m\ddot{\mathbf{r}} = e\mathbf{E}_0 e^{-i\omega t} - \frac{m\ \dot{\mathbf{r}}}{\tau}, \tag{13}$$

where $\tau$ is the mean free time. The Fourier component $\mathbf{r}_\omega$ of electron's coordinate is

$$\mathbf{r}_\omega = i\ \frac{e}{m\ \omega}\ \mathbf{E}_0\ \frac{\tau}{1 - i\ \omega\ \tau}. \tag{14}$$

The dipole momentum $\mathbf{P}$ of the unit volume is equal to $en\mathbf{r}_\omega$ ($n$ is the concentration of electrons), which gives the following expression for the dielectric function

$$\epsilon(\omega) = 1 - \frac{\Omega_L^2}{\omega(\omega + i\ \tau^{-1})}, \quad \Omega_L^2 = \frac{4\pi e^2 n}{m}. \tag{15}$$

Note that the dielectric function is negative at $\omega < \Omega_L$, which means that the fee electron gas is transparent only at very high frequencies $\omega > \Omega_L$. At these frequencies the dielectric function has the following real and imaginary parts:

$$\epsilon_1(\omega) = 1 - \frac{\Omega_L^2}{\omega^2}, \qquad \epsilon_2(\omega) = -i\ \frac{\Omega^2}{\omega^3\ \tau}. \tag{16}$$

In the presence of a static magnetic field $\mathbf{B}$, the Lorentz force $(e/c)\ [\dot{\mathbf{r}}\ \mathbf{B}]$ should be added to the right hand side of Eq (13). This does not affect motion of an electron along the direction of magnetic field (the $z$-axis), but affects the motion in the perpendicular $x, y$-plane. Introducing circular coordinates $r_{\pm} = x \pm iy$, we obtain:

$$r_{\pm} = i \, \frac{e}{m \, \omega} \mathbf{E}_{\pm} \, \frac{\tau}{1 - i \, (\omega \pm \omega_c) \, \tau}; \quad \omega_c = \frac{e \, B}{m \, c}. \tag{17}$$

This gives the asymptotic law of Faraday effect at high frequencies:

$$\epsilon_{xy} - \epsilon_{yx} = \frac{2 \, \Omega_L^2 \, \omega_c}{\omega^3} \tag{18}$$

## Light Scattering by Free Electrons

As it may be seen from Eq (14), the incident light brings about oscillations of the dipole momentum $\mathbf{d}_\omega = e\mathbf{r}_\omega$ of every electron. These oscillations lead to the radiation of light with momentum $\mathbf{k}'$ and intensity

$$dw(\mathbf{k}') = \frac{|\ddot{\mathbf{d}}_\omega \mathbf{e}'|^2}{4\pi c^3} \, dO = \frac{|\mathbf{e}\mathbf{e}'|^2}{4\pi} \, c \, E_0^2 \, r_{cl}^2 \, dO, \qquad r_{cl} = \frac{e^2}{m \, c^2}.$$

where $\mathbf{e}$ and $\mathbf{e}'$ are the polarizations of incident and scattered light respectively. The wave vector $\mathbf{k}'$ of the scattered light lies in same plane as $\mathbf{e}$ and $\mathbf{e}'$. Since the Pointing vector of the incident light is $\Pi = cE_0^2/4\pi$, the effective cross-section of light scattering by a free electron is:

$$\frac{d\sigma}{dO} = r_{cl}^2 \, |\mathbf{e}\mathbf{e}'|^2, \qquad \sigma = \frac{8\pi}{3} \, r_{cl}^2. \tag{19}$$

## Light Absorption in Semiconductors near the Fundamental Edge

Consider a direct-gap semiconductor in the presence of the vector-potential of a monochromatic light $\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_0 \, e^{i\mathbf{kr} - i\omega t}$. The

Hamiltonian of an electron in a periodic potential $U(\mathbf{r})$ and the vector-potential of light has the following standard form:

$$\mathcal{H} = \frac{1}{2m}\left[-i\hbar\nabla - \frac{e}{c}\,\mathbf{A}\right]^2 + U(\mathbf{r}). \tag{20}$$

It is convenient to choose the transverse gauge div$\mathbf{A} \propto \mathbf{k}\mathbf{A}_0 = 0$, $\quad \mathbf{A}_0 = -i(c/\omega)\mathbf{E}_0$. This allows to rewrite the Hamiltonian in the form:

$$\mathcal{H} = -\frac{\hbar^2}{2m}\nabla^2 - i\,\frac{e\hbar}{cm}\mathbf{A}\nabla + \frac{e^2}{2mc^2}\,\mathbf{A}^2 + U(\mathbf{r}). \tag{21}$$

The ratio of the third term on the right hand side of Eq (21) to the second one is of the order of

$$\frac{e}{c}\,\frac{A_0}{p} \sim \frac{eE_0}{\omega}\,\frac{1}{p},$$

i.e. the ratio of an extra momentum, generated by the electric field of light over the period of oscillations, to a thermal (or even atomic) value of momentum. It is a small ratio. Therefore, the third term may be neglected. As a result, the interaction with light is described by the perturbation

$$\mathcal{H}' = \frac{e}{cm}\mathbf{A}\mathbf{p} = -i\,\frac{e\hbar}{cm}\mathbf{A}\nabla. \tag{22}$$

This perturbation causes transitions between different states in the spectrum. The quantum numbers $s$ and $\mathbf{p}$ and the corresponding energies $\epsilon_s(\mathbf{p})$ obey the energy and quasi-momentum conservation laws:

$$\mathbf{p} = \mathbf{p}' + \hbar\mathbf{k}; \quad \epsilon_s(\mathbf{p}) = \epsilon_{s'}(\mathbf{p}') + \hbar\omega. \tag{23}$$

Since $\omega = ck$, where the speed of light $c$ is much larger, than the speed of electrons, the characteristic momentum of light

$\hbar\mathbf{k}$ is much smaller than any relevant momenta of electrons. Therefore, one may neglect the momentum of light and assume that

$$\mathbf{p} = \mathbf{p}'; \quad \epsilon_s(\mathbf{p}) = \epsilon_{s'}(\mathbf{p}) + \hbar\omega. \tag{24}$$

A typical scheme of transitions in a direct gap semiconductor is shown in Fig 1.

Assuming that all states in the valence band are occupied and all states in the conduction band are empty, we find the rate of the inter-band transition $W$ per unit time, using the Fermi golden rule:

$$W = \frac{2\pi}{\hbar}\,\frac{e^2}{m^2\omega^2}\int (d\mathbf{p})\ |\mathbf{E}_0\ \mathbf{p}_{vc}(\mathbf{p})|^2\ \delta(\epsilon_c(\mathbf{p}) - \epsilon_v(\mathbf{p}') - \hbar\omega), \tag{25}$$

where the matrix element of the momentum is

$$\mathbf{p}_{vc}(\mathbf{p}) = \int d\mathbf{r}\ \psi^*_{c\mathbf{p}}(\mathbf{r})\ \hat{\mathbf{p}}\ \psi_{v\mathbf{p}}(\mathbf{r}) = -i\hbar\int d\mathbf{r}\ u^*_{c\mathbf{p}}(\mathbf{r})\ \nabla\ u_{v\mathbf{p}}(\mathbf{r}) \tag{26}$$

The absorption coefficient $\alpha(\omega)$ may be found if the rate of transition is divided by the number of incident light quanta $N$ per unit time. Since $N = (c/4\pi)E_0^2/n\hbar\omega$ ($n$ is the refraction index), the absorption coefficient for light with polarization $\mathbf{e}$ is equal to:

$$\alpha(\omega) = \frac{e^2}{\pi m^2 c n\omega}\int (d\mathbf{p})\ |\mathbf{e}\ \mathbf{p}_{vc}(\mathbf{p})|^2\ \delta(\epsilon_c(\mathbf{p}) - \epsilon_v(\mathbf{p}) - \hbar\omega), \tag{27}$$

Assume that the energy minimum in the conduction band and the energy maximum in the valence band both correspond to a certain point in the Brillouin zone $\mathbf{p} = \mathbf{p}_0$ [1]. It follows from Eq (27), that the rate of transition vanishes if $\hbar\omega < E_g =$

[1] This occurs in semiconductors of $A_4B_6$-group with important examples of $PbS$, $PbSe$, $PbTe$ and $SnTe$.

$\epsilon_c(\mathbf{p}_0) - \epsilon_v(\mathbf{p}_0)$. This establishes the energy gap $E_g$ as a threshold, the fundamental edge of light absorption. Near the fundamental edge, the essential values of quasi-momentum are $\mathbf{p} \approx \mathbf{p}_0$. If the matrix element at the point $\mathbf{p} = \mathbf{p}_0$ does not vanish $(\mathbf{p}_{vc}(\mathbf{p}_0) \neq 0$, it is a *permitted transition*, otherwise it is a *forbidden transition.* The absorption coefficient for a permitted transition is, therefore,

$$\alpha_{perm}(\omega) = \frac{4\sqrt{2}\, e^2}{m^2 c n \omega} \sum_{\mathbf{p}_0} |\mathbf{e}\ \mathbf{p}_{vc}(\mathbf{p}_0)|^2\ \frac{\sqrt{\mu_1 \mu_2 \mu_3}}{\hbar^3} \sqrt{\hbar\omega - E_g}, \quad (28)$$

where the reciprocal masses $\mu_i$ along the principal axes are introduced

$$\frac{1}{\mu_i} = \frac{1}{m_{ci}} + \frac{1}{m_{vi}}$$

The sum over the extrema $\mathbf{p}_0$ gives the dependence of the absorption on light polarization. In the case of a cubic crystal, this sum gives just a constant, which means that there is no anisotropy. It is important to notice that permitted light absorption is accompanied by generation of free electrons and free holes with essentially equal populations for different directions of their momenta.

In the case, when the extremum occurs at centre of the Brillouin zone $\mathbf{p}_0 = 0$,

$$\alpha_{perm}(\omega) = \frac{2\, e^2}{m^2 c n \omega}\ |\mathbf{e}\ \mathbf{p}_{vc}(0)|^2\ \left(\frac{2\mu}{\hbar}\right)^{3/2} \sqrt{\hbar\omega - E_g}, \quad (29)$$

In the case of a forbidden transition with $\mathbf{p}_0 = 0$,

$$|\mathbf{e}\ \mathbf{p}_{vc}(\mathbf{p})|^2 = e_\alpha\ e^*_\beta\ d_{\alpha\gamma}\ d^*_{\beta\delta}\ p_\gamma\ p_\delta, \qquad d_{\alpha\beta} = \left(\frac{\partial p^\alpha_{vc}}{\partial p_\beta}\right)_{p=0}.$$

In a cubic crystal

$$d_{\alpha\beta} = d\ \delta_{\alpha\beta}, \qquad |\mathbf{ep}_{vc}|^2 = d^2(\mathbf{ep})^2, \tag{30}$$

and the absorption coefficient is equal to

$$\alpha_{forb}(\omega) = \frac{2\ e^2 d^2}{3m^2 cn\omega}\ \left(\frac{2\mu}{\hbar}\right)^{5/2} (\hbar\omega - E_g)^{3/2}. \tag{31}$$

Eq (30) shows that forbidden light absorption leads to generation of electrons and holes with a fairly anisotropic distribution function $f(\mathbf{p}) \propto \cos^2\theta$, where $\theta$ is the angle between the direction of the momentum $\mathbf{p}$ of a photo-generated electron and the linear polarization of light $\mathbf{e}$.
Imaginary part of the dielectric function $\epsilon_2(\omega)$ may be expressed through the absorption coefficient

$$\epsilon_2(\omega) = \frac{nc}{\omega}\ \alpha(\omega) \tag{32}$$

Using Eqs (29, 31), one may see that the imaginary part of the dielectric function $\epsilon_2(\omega)$ does not depend on the refraction index $n$.

## Light Absorption in Semiconductors near the Fundamental Edge. Interaction in the Final State. Excitons

The first order perturbation treatment of light absorption is valid, because of weak coupling of electrons with light. There is another type of electromagnetic interaction, which plays an important role in light absorption. It is the Coulomb interaction between the electron and the hole generated by light. The characteristic energy Ry may be estimated as either the mean kinetic energy $T \sim \hbar/\mu\tilde{a}_B^2$, or the mean potential energy $|U| \sim e^2/\kappa\tilde{a}_B$ (here $\tilde{a}_B$ is the effective distance between the electron and the hole, $\kappa$ is the dielectric function at relevant frequencies, and $\mu$ is the reduced mass of the electron and the hole, which is expressed through the effective masses of these particles as $\mu = m_e^* m_h^*/(m_e^* + m_h^*)$). Assuming that the mean potential energy and the mean kinetic energy at the separation $\tilde{a}_B$ are of the same order, we may find that

$$|U| \sim \frac{e^2}{\kappa a_B} \sim T \sim \frac{\hbar^2}{\mu\tilde{a}_B^2} \to \tilde{a}_B \sim \frac{\hbar^2\kappa}{\mu e^2}, \quad \mathrm{Ry} \sim T \sim \frac{\mu e^4}{\kappa^2}. \qquad (33)$$

Thus, if the energy of the photon $\hbar\omega$ is much higher than the fundamental edge $Eg$ ( $(\hbar\omega - E_g) \gg \mathrm{Ry}$ ), then the interaction between the photo-generated electron and hole may be neglected and they may be considered as freely moving particles. In the close vicinity to the fundamental edge the mentioned interaction is essential[2]. Practically, this means that instead of the plane waves, the matrix element in Eq (26) should be calculated with the exact wave function $\psi(\mathbf{r})$ for the relative motion of the electron and the hole, which obeys the equation

$$\left\{-\frac{\hbar^2}{2\mu}\,\nabla^2 - \frac{e^2}{\kappa r} + E_g - \hbar\omega\right\}\psi = 0. \qquad (34)$$

[2]Due to large polarizability $\kappa \gg 1$ and a small effective mass of, at least, one of the carriers $\mu \ll m_e$, the effective Bohr radius $\tilde{a}_B$ is significantly larger, than the inter-atomic distance, while the effective energy scale Ry is not only smaller than the atomic energy but even than the relatively small energy of forbidden gap, $E_g$. This means that the processes of interaction of photo-generated carriers plays a significant role only at the energies close to the bottom of the conduction and the top of the valence band.

The spectrum of the Hamiltonian (34) contains an infinite number of bound states with the binding energy $E_\nu$ dependent on the principal quantum number $1 \leq \nu < \infty$

$$E_\nu = \hbar\omega - E_g = -\ \frac{\mathrm{Ry}}{\nu^2}; \quad \mathrm{Ry} = \frac{\mu e^4}{2\kappa^2}. \tag{35}$$

These states correspond to the zero momentum ($\mathbf{p} = 0$) states of new modes of neutral excitations (excitons) with the energies

$$E_\nu(\mathbf{p}) = E_\nu + \frac{\mathbf{p}^2}{2M}, \tag{36}$$

where the total mass of the exciton is equal to the sum of effective masses of the electron and the hole $M = m_*^{(c)} + m_*^{(v)}$. The excitons appear as bright resonances at the frequencies below the fundamental edge of absorption. They are most important for photo-luminescence: if the electron-hole pairs are generated at the energies well above the threshold, they lose their energies due to emission the phonons and finally form the exciton in its ground state. So, the exciton emission line dominates the luminescence.

The Coulomb interaction of the photo-generated electrons and holes also changes the wave function of their relative motion in the continuous spectrum. giving rise to an extra-factor:

$$\frac{|\psi(0)|^2}{|\psi_0(\mathbf{p})|^2} = \frac{\beta e^\beta}{\sinh\beta}, \qquad \beta = \frac{\pi\hbar}{p a_B} = \pi\ \left(\frac{2\mu}{\hbar^2}\right)^{1/2}\ \frac{\hbar}{p}\ \mathrm{Ry}^{1/2} \tag{37}$$

in Eq (27). As a result, at frequencies $\omega$ very close to the fundamental edge, Eq (29) and (30) take the following form:

$$\alpha_{perm}(\omega) = \frac{2\pi e^2}{m^2 c n \omega}\ |\mathbf{e}\ \mathbf{p}_{vc}(0)|^2\ \left(\frac{2\mu}{\hbar}\right)^{3/2} \sqrt{\mathrm{Ry}}\ \frac{e^z}{\sinh z}, \tag{38}$$

$$\alpha_{forb}(\omega) = \frac{2\ e^2 d^2}{3m^2 cn\omega} \left(\frac{2\mu}{\hbar}\right)^{5/2} \mathrm{Ry}^{3/2} \left(1 + \frac{\pi^2}{z^2}\right) \frac{e^z}{\sinh z}, \quad (39)$$

where

$$z = \pi \sqrt{\frac{\mathrm{Ry}}{\hbar\omega - E_g}} \quad (40)$$

All this leads to a strong renormalization of the light adsorption rate at the fundamental edge:

$$\alpha_{perm}(\hbar\omega = E_g) = \frac{4\pi e^2}{m^2 cn\omega} \ |\mathbf{e}\ \mathbf{p}_{vc}(0)|^2 \left(\frac{2\mu}{\hbar}\right)^{3/2} \sqrt{\mathrm{Ry}}, \quad (41)$$

$$\alpha_{forb}(\hbar\omega = E_g) = \frac{4\pi\ e^2 d^2}{3m^2 cn\omega} \left(\frac{2\mu}{\hbar}\right)^{5/2} \mathrm{Ry}^{3/2}. \quad (42)$$

The intensities of the absorption lines at frequency $\omega$, which corresponds to the exciton state with the principal quantum number $\nu$, equal

$$\alpha_{perm}(\omega) = \frac{4\pi}{nc} \frac{e^2 E_g}{\hbar\mu a_B^3} \sum_{\nu=1}^{\infty} \frac{1}{\nu^3}\ \delta(\hbar\omega - E_g + E_\nu), \quad (43)$$

$$\alpha_{forb}(\omega) = \frac{16\pi\theta^2}{3cn} \frac{e^2\hbar}{a_B^3 \mu\omega} \sum_{\nu=1}^{\infty} \frac{\nu^2 - 1}{\nu^5}\ \delta(\hbar\omega - E_g + E_\nu). \quad (44)$$

Here $\theta$ is a dimensionless numerical factor. Note that if the inter-band transition is forbidden then the exciton series begins with the first excited state of the exciton ($\nu \geq 2$).
If the excitation is very strong and the concentration of the photo-generated electrons and holes is high, their cooling down may give rise to multi-exciton configurations; i.e. either exciton-impurity complexes, or the two-exciton molecules ( bi-excitons ). These multi-exciton complexes give characteristic lines in the luminescence spectrum.

## Indirect Transitions

In a number of important examples, the energy minima in the conduction band and the energy maxima in the valence band are located at different points of the Brillouin zone[3]. Under this condition, the optical transitions with the identical quasi-momentums of the electron and the hole have a larger threshold $E_o$, than the thermal gap $E_g = \epsilon_c(\mathbf{p}_0) - \epsilon_v(0)$. The emission (absorption) of a phonon of the branch $s$ of the vibration spectrum removes the restriction originated in quasi-momentum conservation. As a result, the fundamental edge of absorption moves to $E_g \pm \hbar\omega_s(\mathbf{p}_0)$, i.e. nearly to the thermal gap. In the frequency interval $E_g < \hbar\omega < E_o$ for the case of permitted transition, the adsorbtion coefficient behaves as (see problems):

$$\alpha_{perm}^{indir} = \frac{B_{emit}(\hbar\omega - E_g - \hbar\omega_s(\mathbf{p}_0))^2 + B_{abs}(\hbar\omega - E_g + \hbar\omega_s(\mathbf{p}_0))^2}{\omega(E_o - \omega)^2}. \tag{45}$$

Therefore, in the case of indirect gap in the energy spectrum of a semiconductor, the adsorption rises relatively slowly at the frequencies near the fundamental edge. After the frequency reaches the threshold for the direct transition, the absorption starts to rise more rapidly with the frequency, as it is shown in Fig 2.
Similar to the situation with the light absorption in semiconductors with direct forbidden gap, the interaction of the photo-generated electron and hole, plays an important role in adsorption in indirect semiconductors. The excitons in indirect semiconductors live longer, because the electron and the hole need for their recombination an interaction with an agent (a phonon), which would take away a large momentum. This allows to pump up the exciton gas in these semiconductors to a pretty high concentration and study collective phenomena.

## Polaritons

Space dispersion of dielectric function $\epsilon(\omega, \mathbf{k})$ plays a very important role in electrodynamics near the exciton absorption line at $\omega = \omega_0$. It results in a very peculiar spectrum of elementary excitations. Consider this problem in the simplest case

[3]For instance, in $Ge$ and $Si$ the conduction band has many minima (four for $Ge$ and six for $Si$) positioned at the points $\mathbf{p}_i \neq 0$, while the valence band has a singe maximum at the center of the Brillouin zone $\mathbf{p} = 0$.

of isotropic medium. Thus, let, first, the medium has an inversion center. Then, dielectric function $\hat{\epsilon}(\omega, \mathbf{k})$ may be split into longitudinal and transverse functions $\epsilon_l$ and $\epsilon_{tr}$ respectively:

$$\epsilon_{\alpha\beta}(\omega, \mathbf{k}) = \epsilon_l(\omega, \mathbf{k}) \ \frac{k_\alpha k_\beta}{k^2} + \epsilon_{tr}(\omega, \mathbf{k}) \ \left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right). \tag{46}$$

We are interested in light absorption and, therefore, predominantly, in transverse part of dielectric function. Near the absorption line it has the following form:

$$\epsilon_{tr}(\omega, \mathbf{k}) = a + \frac{A}{\omega_t(\mathbf{k}) - \omega}, \qquad \omega_t(\mathbf{k}) = \omega_0 + \gamma k^2, \quad A > 0. \tag{47}$$

At zero wave vector $\mathbf{k} = 0$ the longitudinal dielectric function $\epsilon_l$ coincides with the transverse one.

$$\epsilon_l(\omega, 0) = a + \frac{A}{\omega_0 - \omega}. \tag{48}$$

A longitudinal eigen-mode $\omega_l(\mathbf{k})$ is determined by equation $\epsilon_l(\omega_l(\mathbf{k}), \mathbf{k}) = 0$. Eq (48) gives, therefore,

$$\omega_l(0) = \omega_0 + \frac{A}{a}. \tag{49}$$

The dispersion relation for light in a medium has the following form:

$$\det \left| \epsilon_{\alpha\beta}(\omega, \mathbf{k}) \ \omega^2 - c^2 \left(k^2 \delta_{\alpha\beta} - k_\alpha k_\beta\right) \right| = 0, \tag{50}$$

which, after the substitution $k = n\omega/c$ takes the form of an equation

$$(n^2 - a)\left(\beta \ n^2 - \omega + \omega_0\right) = A, \quad \beta = \frac{\gamma \ \omega_0^2}{c^2}. \tag{51}$$

which couples the refraction index $n$ with the frequency $\omega$. Solutions of this equation for the positive and negative values of dispersion coefficients $\beta$ are presented at Fig 3. a and b respectively. One may see that for $\beta > 0$ (Fig 3a) Eq (51) has one solution with $n^2 > 0$ at $\omega \le \omega_l(0)$ and two solutions at $\omega \ge \omega_l(0)$. For $\beta < 0$ (Fig 3 b) Eq (51) has either one, or two, or even no solutions at all. In the frequency range, where Eq (51) has no real solutions $n(\omega)$, it apparently has complex solutions, which means that this frequency range corresponds to total reflection.
In the case, when the medium has no inversion center, its dielectric function has space dispersion due to gyrotropy. The electric field $\mathbf{E}$ may be expressed through displacement $\mathbf{D}$ as

$$\mathbf{E} = \frac{1}{\epsilon(\omega)} \ \mathbf{D} + iG \ [\mathbf{D} \ \mathbf{n}], \quad \epsilon(\omega) = \frac{A}{\omega_0 - \omega}, \tag{52}$$

and the dispersion relation takes the form:

$$\left(\frac{1}{n^2} + \frac{\omega - \omega_0}{A}\right)^2 = G^2 n^2. \tag{53}$$

Fig 4 shows solutions $n(\omega)$ of Eq (53) at frequency $\omega$ close to that of absorption line $\omega_0$. One may see from Fig 4 that Eq (53) has either one or three solutions. Appearance of extra-waves in a medium near absorption line is a signature of Polaritons, bound states of excitons and light.